

**CONCOURS D'ENTREE EN 1ère ANNEE – SESSION DE JUILLET 2018**

**EPREUVE DE MATHEMATIQUES**

**Durée 3h00 - Coefficient 4**

**EXERCISE 1** : 5 Marks

One carries out tests on sample of 220 flashlights in order to test their lifetime. This duration is expressed in hours. The results are gathered by classes of amplitude equal to 100 hours in the following table:

Duration in thousands	[1,1 ; 1,2]	[1,2 ; 1,3]	[1,3 ; 1,4]	[1,4 ; 1,5]	[1,5 ; 1,6]	[1,6 ; 1,7]	[1,7 ; 1,8]	[1,8 ; 1,9]
Effective	6	14	25	75	80	10	8	2
increasing cumulative Frequencies								

It is supposed that the distribution is uniform inside each class.

- 1.a) Trace the histogram of this series. 1pt
  - b) Recopy and supplement the table above. 0,75pt
  - c) Calculate the approximate value of the median with  $10^{-1}$  close by default. 0,5pt
2. a) Trace the polygon of the increasing cumulative frequencies of this series. 0,75pt
  - b) Calculate truncations of order 1 of the average and the standard deviation  $\sigma$  of the series. 1pt
3. Which is the percentage of lamps of which the lifetime is in  $[\bar{x} - \sigma ; \bar{x} + \sigma]$  0.5pt
4. Another batch of flashlights of the same power coming from another manufacturing is also tested. The average of lifetime is of 140. Which is that of the two batches which seems better to you? 0,5pt

**EXERCISE 2: 5 Marks**

One launches five times of continuation a die whose faces are numbered from 1 to 6.

Determine the probability of the following events:

- a) All the figures are lower or equal to 2. 0,5pt
- b) figure 6 figure exactly three consecutive times. 0,75pt
- c) Where figure at least twice figure 1. 0,75pt

II E indicates an Euclidean vectorial plan, provided with a direct base orthonormal  $B = (\vec{i}; \vec{j})$ .

Ballot box A contains five tokens of which two increase number 0, one the number  $\frac{1}{2}$ , one the number -1 and one the number  $\frac{\sqrt{2}}{2}$ . The ballot box B contains four tokens of which two increase number 1, one the number  $-\frac{\sqrt{2}}{2}$  and one the  $\frac{\sqrt{3}}{2}$  number. One draws from these two ballot boxes, two tokens, one of ballot box A, the other one of the ballot box B. One indicates a has the number carried by the token drawn from A, and by b the number carried by the token drawn from B and  $f_{a,b}$  the endomorphism of E whose matrix compared to the base B is form  $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$

- 1 - Calculate the probability so that  $f_{a,b}$  is a vectorial rotation. 1pt
- 2 - Calculate the probability so that  $f_{a,b}$  is a vectorial rotation of right angle. 1pt
- 3 - One carries out three successive pullings with handing-over. Calculate the probability of obtaining at least a vectorial rotation. 1pt

**EXERCISE 3 5 Marks**

The plan is provided with an orthonormal reference mark (O, I, J). One considers the function f defined by:

$$\begin{cases} f(x) = x - 1 + \frac{1}{x}, si \ x \leq 1 \\ f(x) = 1 - (\ln x)^2, si \ x > 1 \end{cases}$$

- 1-a) Show that f is continuous and derivable into 1. 1pt

b) Calculate the limits of  $f$  at the boundaries of definition set and specify the infinite branches of the curve representative (C) of  $f$ . 0,75pt

c) Study the variations of  $f$  then show that the point of X-coordinate  $e$  base napierian logarithm is a point of inflection of (C). 1pt

d) Trace (C). 0,75pt

2)  $h$  is the restriction of  $f$  on the interval  $]1; +\infty[$ .

a) Show that  $h$  carries out a bijection from  $]1; +\infty[$  towards an interval which one will determine 0,5pt

b) Deduce that  $h$  admits a reciprocal function  $h^{-1}$  whose the direction of variation will specify. Then build the curve representative of  $h^{-1}$ . 1pt

**EXERCICE 4 : 5 Marks**

$E$  is a real euclidean vector space,  $(\vec{i}; \vec{j})$  a base orthonormal of  $E$ ,  $\vec{u}_0$  a vector of  $E$  of components  $(\alpha, \beta)$ , has being a given real number, one defines the linear application  $f_a$  of  $E$  in  $E$  by:  $\vec{u} \in E, f_a(\vec{u}) = \vec{u} + a(\vec{u} \cdot \vec{u}_0)\vec{u}_0$

1-a)  $\vec{u}$  is a vector of  $E$  of components  $(x, y)$ . Calculate  $\vec{u} \cdot \vec{u}_0$ . 0,5pt

b) Calculate the components  $(x', y')$  of  $f_a(\vec{u})$  according to the components  $(x, y)$  of  $\vec{u}$  and  $\alpha, a$  and  $\beta$ . 0,5pt

2 –  $g_a$  is the endomorphism of  $E$  which with any vector  $\vec{u}(x, y)$  associates the vector  $g_a(\vec{u})$  components  $(x', y')$  defined by:

$$\begin{cases} x' = (1 + 4a)x + 2ay \\ y' = 2ax + (1 + a)y \end{cases}$$

a) Calculate  $g_a(\vec{i})$  and  $g_a(\vec{j})$ . 1pt

b) Give the  $g_a$  matrix in the base  $(\vec{i}; \vec{j})$ . 0,5pt

c) For which value of  $a$   $g_a$  is an automorphism? In each case deduce  $\text{Ker } g_a$  and  $\text{Im } g_a$ . 0,75pt

d) For which value of  $a$   $g_a$  is not an automorphism? Under this condition determine  $\text{Ker } g_a$  and  $\text{Im } g_a$ . 1,25pt